

The recommendation for selecting the rib thickness, as shown by the analysis, is connected with the particular value of the rib pitch. This is explained by the fact that the value of the thickness depends on the pitch at which the construction costs of the heat exchanger are at a minimum.

Thus, the relationships given in the present paper for calculating the heat exchanger efficiently make it possible to select the parameters of standardized heat pipes which ensure a minimum cost for the heat exchanger construction. Simultaneously the arrangement of the heat pipes is determined which satisfies the requirements with respect to the efficiency of the heat exchanger and the permissible pressure drop in the heat exchange layout. In this case the procedure for the design calculations includes the following steps: the selection from the working conditions of the working fluid for the heat pipes the maximum possible length for the heat pipes, and the pitch of the ribbing; setting up for various ratios of the internal to the external diameters of the ribs relationships between the cost of constructing the heat exchanger and the diameter of the tubes supporting the ribs; and for the diameter selected, determining the rib heights, then the rib thickness and the spacing between the ribs.

#### NOTATION

$l_e$ , length of the evaporation zone of the heat pipe;  $l_c$ , length of the condensing zone of the heat pipe;  $d$ , diameter of pipe supporting ribs;  $d_{pi}$ , internal diameter of pipe;  $\epsilon$ , ribbing coefficient;  $\alpha'_{out}$ ,  $\alpha''_{out}$ , external heat-transfer coefficients in the evaporation and condensation zones, respectively;  $\alpha_e$ , heat-transfer coefficient in evaporator;  $\alpha_c$ , heat-transfer coefficient in condenser;  $\lambda$ , thermal conductivity of shell of heat pipe and its ribbing;  $P$ , weight of heat pipe;  $C_{TT}$ , manufacturing cost of an individual heat pipe;  $C_M$ , cost of unit weight of the shell material.

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#### LIMITS OF APPLICABILITY OF THE THEORY OF A SINGLE-PHASE BOUNDARY LAYER USING CONDITIONS OF FREE-CONVECTIVE FROSTING

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The conditions of applicability of the approximation of a single-phase boundary layer with the formation of a condensate in the form of frost on a vertical flat surface are determined.

I know of no published studies of the limits of applicability of the theory of a single-phase boundary layer in the analysis of heat and mass transfer under conditions of precipitation of a solid condensate (frost) from a gaseous binary mixture (containing predominately inert components) on an isothermal surface. One of the main reasons for this is the uncritical use of correlations for the Nusselt and Sherwood numbers. The analytical basis for them is the theory of a single-phase (multicomponent) boundary layer, in spite of the difference in the temperatures (medium-isothermal surface) under which it is formed. Thus, for example, almost all theoretical approaches to the determination of the coefficient of heat and mass transfer under the conditions of formation of frost from moist air on isothermal surfaces were constructed for single-phase models [1]. On the other hand, in many experiments it was found that the water phase changes spontaneously even before the frost layer forms on it [2].

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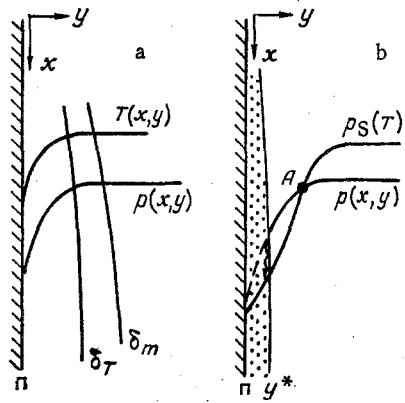


Fig. 1

Fig. 1. Profiles of the temperature and pressure in the boundary layer: a) single-phase boundary layer; b) fog formation in the boundary layer.

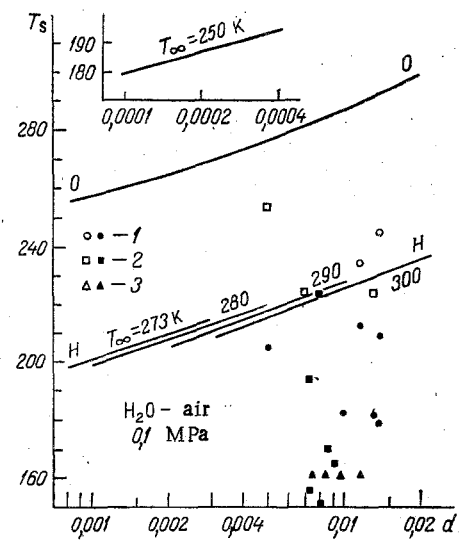


Fig. 2

Fig. 2. Chart of demarcations of the single- and multi-phase free-convective boundary layers: 1) based on [2]; 2) [7]; 3) [6]; the dark points indicate the appearance of a fog sublayer and the light-colored points indicate the absence of such a sublayer; 00) thermodynamic equilibrium; HH) critical metastability.  $T_s$ , K.

The lack of an accurate description of the spontaneous change in phase of the component that condenses in a real boundary layer was the primary reason for the lack of the required theoretical model, with whose help the region of the boundary layer with one phase and the region with a large number of phases could be demarcated. The appearance of a large number of phases does not correspond to the model of the "single-phase" theory. For this reason, in this work, the appearance of a spontaneous change in phase in an initially single-phase boundary layer (after which it becomes multiphase) is employed as the criterion for the inapplicability of the theory of a single-phase boundary layer.

The phase transformation of the condensing component is described by the classical theory of homogeneous heterophase fluctuations [3, 4], which are included within the framework of the initial, multiphase boundary layer. Combining the theory of homogeneous heterophase fluctuations and the theory of a multiphase free-convective boundary layer enables indicating the collection of parameters that determine the conditions for inapplicability (or, conversely, applicability) of the theory of a single-phase boundary layer.

We shall first examine the reason for the spontaneous change in phase accompanying condensation in a boundary layer.

The profiles of the temperature and partial pressures in a free-convective monophasic multicomponent boundary layer under the conditions of simultaneous heat and mass transfer to an isothermal surface obey the standard model, which consists of a system of differential equations (continuity, angular momentum, energy, and mass) and the corresponding boundary conditions [5]. These profiles can be determined by standard methods from this system of equations, for example, in the form:

$$\Theta = \left(1 - \frac{y}{\delta_T}\right)^n, \tag{1}$$

$$\Theta_D = \left(1 - \frac{y}{\delta_m}\right)^m. \tag{2}$$

Figure 1 shows schematically two profiles: the temperature level in the surrounding medium drops to the temperature level on the isothermal surface in accordance with Eq. (1) (Fig. 1a). An analogous situation is also observed with the partial pressure of the condensing component.

Let us see what would happen with the equilibrium partial pressure of the condensing component, if the temperature of the isothermal surface were so low that a new phase (fog) could appear in the boundary layer. In accordance with the temperature profile the equilibrium partial pressure drops uniformly and at a certain moment reaches the partial pressure of the condensing component (the point A in Fig. 1b) and then, closer to the surface, drops below this pressure. Hence in a monophasic boundary layer the condensing component transforms into the metastable state, which corresponds to local supersaturation. The degree of this supersaturation can be expressed in the form

$$S = p/p_s \quad (3)$$

According to the classical theory of homogeneous heterophase fluctuations, the nuclei of a new phase can form in the metastable condensing component appearing in this manner, and in addition the number of nuclei will be all the larger, the higher the supersaturation is. The so-called critical partial pressure of the condensing component corresponds to critical supersaturation, and these are precisely the conditions under which the number of conglomerates of particles per unit volume and unit time ( $J$ ) corresponds to the population of nuclei ( $J_{cr}$ ) that we define as a macroscopic "fog." We postulate the achievement of these conditions in the boundary layer as the sought criterion for regarding the single-phase boundary layer as a multiphase layer.

This model of a spontaneous phase change in the boundary layer, unlike the well-known models of [1, 5, 6], simultaneously satisfies the given geometry of the boundary layer, the system of differential equations of the multiphase boundary layer, and the additional condition

$$p(x, y)|_{y=y^*} = p_n(T) S [J_{cr}, T(x, y^*)], \quad (4)$$

obtained from the theory of homogeneous heterophase fluctuations, so that the existence of a self-similar boundary of the fog sublayer is established (see Fig. 1b), which, in any case, indicates that further application of the theory of a single-phase boundary layer is not correct.

There naturally arises the question of what practical result can be obtained from this model in order to demarcate the applicability of the theory of a single-phase boundary layer for large temperature differentials and the expected metastability of the condensing component? The answer to this question based on the proposed model and its mathematical description is quite simple. The condition posed for the existence of a spontaneous phase change by the mechanism of homogeneous heterophase fluctuations corresponds uniquely to the collection of parameters consisting of the temperature of the isothermal surface  $T_s$ , the temperature in the medium outside the boundary layer  $T_\infty$ , and the content of the condensing component  $d_\infty$  in the medium, for which the given condition holds. If it were possible to interrelate these quantities based on the described model, a "map of demarcations of the single- and multiphase free-convective boundary layers" (briefly, the "map of demarcations"), suitable for practical applications, would be obtained.

Figure 2 shows the interrelationship of these quantities, obtained for the water-air system (moist air at 1 bar). The temperature of the isothermal surface is plotted along the ordinate axis and the content of the condensing component is plotted along the abscissa axis; the temperature of the medium  $T_\infty > T_s$ . There are three regions in the diagram: the first region lies above the line 00, the second lies between the line 00 and the system of lines HH, and the third lies below the lines HH. Any point above the line 00 is determined by the system of values of  $(T_s, d_\infty)$  for any  $T_\infty > T_s$ , for which a phase change is thermodynamically impossible anywhere in the boundary layer. The points on the line 00 correspond to the relationship between the temperature and the moisture content in the air under the conditions of thermodynamic equilibrium. Hence, if the temperature of the isothermal surface is higher than the equilibrium temperature for the given moisture content in the medium (the line 00), a phase change does not occur in the boundary layer. Under the conditions of homogeneity of an inert component (in this case air) the phase transition is realized exclusively on the isothermal surface (heterogeneous and heterophase fluctuations). The condensate is either a liquid (dew) or solid (frost), depending on the temperature of the surface. In the other region below the line 00 the temperature of the isothermal surface is below equilibrium, i.e., for the given moisture content in moist air the boundary layer in the sublayer with supersaturated condensing component (see Fig. 1) becomes metastable, but in accordance with the described model the increment to the nuclei of the new phase is still not large enough for a stable fog, visible under a microscope, to form. If, however, the temperature of the isothermal surface for the given values of the moisture content and temperature in air reaches

a level on the corresponding line HH within the boundary layer, the obtained metastability of the condensing component on the self-similar boundary of the future fog sublayer (see Fig. 1b) becomes critical. In the boundary layer a fog sublayer forms, i.e., the boundary layer becomes multiphase. If, however, the temperature of the isothermal surface is still low, then we penetrate even more deeply into the region of homogeneous heterophase fluctuations, and the theory of a single-phase boundary layer is no longer applicable.

The results obtained are a consequence of theoretical analysis. There arises the question of how well this theory agrees with experiment. It must be emphasized that in the literature there are virtually no systematic experimental studies of a spontaneous phase change in a boundary layer with strict control of, especially, the homogeneity of the inert component. But, based on rare observations, it is possible to check for the first time the theory founded in this manner. Figure 2 shows the results of some experimental observations of the appearance of a fog sublayer in a boundary layer (or its absence). The results obtained do not disagree with the theoretical data, but they are not sufficient for full confirmation of the proposed criteria.

#### NOTATION

d, moisture content of the air; J, number of conglomerates per unit volume per unit time,  $m^{-3} \text{ sec}^{-1}$ ; m, n, parameters in Eqs. (1) and (2); p, pressure, Pa; S, degree of supersaturation of water vapor; T, temperature, K; x and y, distances measured along the x and y directions, respectively, in m;  $\delta$ , thickness in m;  $\theta$  and  $\theta_D$ , dimensionless temperature and concentration of the water vapor. Indices: T, temperature; m, concentration boundary layer; S, saturated vapor; cr, critical; \*, fog;  $\infty$ , corresponds to a quantity evaluated in the surrounding medium; and s, surface.

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